Limiting reinforcement ratios for RC flexural members

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The minimum and maximum limits on longitudinal and transverse reinforcement ratios provided for reinforced concrete flexural members in the Indian code is based on tests conducted on normal strength concrete, and hence not applicable to high strength concrete beams. Hence comparing the provisions of other national codes, modifications to these limits are proposed for inclusion in the next edition of the code. These modified expressions are necessary in order to prevent sudden and brittle collapse of flexural members and also to provide ductile behaviour.

Keywords: *Ductile behaviour, High strength concrete, Minimum tension reinforcement, Maximum tension reinforcement, Minimum transverse reinforcement, Maximum transverse reinforcement, maximum diameter of bars.*

Minimum and maximum limits on longitudinal and transverse reinforcement ratios are often prescribed in codes of practices for reinforced concrete flexural members. The minimum limit is prescribed to avoid sudden and brittle failure in case of accidental overload, or to take care of additional tensile forces due to shrinkage, temperature, creep or differential settlement. The maximum limit is prescribed to avoid compression failure of concrete before the tension failure of steel, thus ensuring sufficient rotation capacity at ultimate limit state. Similar limits are prescribed on transverse reinforcement, as shear failures are more catastrophic than flexural failures. When shear reinforcement are provided, they restrain the growth of inclined cracking, and increase safety margin against failure. Ductility is also increased and a warning of failure is provided.

Although the Indian code on reinforced concrete, IS 456, was revised in 2000, most of the design provisions in the 1978 version of the code were retained, without modifications.[1] Moreover, most of the provisions in the code are based on experiments conducted on RC elements having strengths up to 40 MPa. In a proposed amendment to this code, BIS has redefined high strength concrete by designating grades up to M60 as standard concrete and grades M65 to M100 as high strength concrete. Thus, the existing provisions are simply extrapolated up to grade M60. Also there are no special provisions for high strength concrete, i.e. for grades M65 to M100. Such extrapolation of rules for normal strength concrete (NSC) to high strength concrete (HSC) may be erroneous as high strength concrete, in spite of enhanced strength and durability, tend to be more brittle than normal-strength concrete, due to its more homogeneous microstructure (In NSC, where the aggregate is stronger than the cement paste, cracks propagate around the aggregate. These longer crack paths consume more energy. In HSC, the aggregates become the weaker part of the matrix. Shorter cracks form through the aggregates using less energy. Thus, propagation of cracks is more sudden and brittle).

Moreover, the minimum and maximum limits on longitudinal and transverse reinforcement in the Indian code depend only on steel strength and are independent of concrete strength. But for HSC it may be prudent to include the concrete strength also in the equation of such limits. Hence, in this paper the Indian code provisions are compared with the latest American code provisions (which have been modified three times after 2000, and hence reflect current state-of-the-art research), and suitable modifications are proposed for the Indian code. It is shown that the provisions in other codes of practices such as Canadian, New Zealand, and Eurocode 2, are also similar to those found in the American code.

Minimum tension reinforcement

The nominal moment of resistance (M_n) of a reinforced concrete beam as shown in Figure 1, with an effective depth d, and breath b is given by the Indian code, using a parabolic-rectangular stress block, as[1]

$$M_n = 0.87 f_y A_{st}\, d \; [1 - \frac{f_y A_{st}}{f_{ck} bd}] \qquad ......(1)$$

where

f_y = Characteristic yield strength of reinforcement

A_{st} = Area of reinforcement

f_{ck} = Characteristic cube compressive strength of concrete

For architectural or other reasons, beams may be provided in larger sizes than required for flexural strength. With a small amount of tensile reinforcement, the computed strength of the member using cracked section analysis (using Equation 1), may become less than that of the corresponding strength of an un-reinforced concrete section, computed using modulus of rupture. This will result in sudden and brittle failure of such beams. To prevent such possibilities, codes of practices often prescribe minimum amount of tension reinforcement. Minimum steel is also provided from shrinkage and creep considerations, which often control the minimum steel requirement of slabs. Minimum steel will also guarantee accidental overloads due to vibration, settlements, etc.

Hence, the required condition for minimum percentage of steel may be stated as

Strength as reinforced concrete beam > Strength as plain concrete beam ......(2)

The value of modulus of rupture (tensile strength) of concrete, f_{cr}, is given by the code as[1]

$$f_{cr} = 0.7 \sqrt{f_{ck}} \qquad ......(3)$$

Hence the moment of resistance for an unreinforced concrete beam, M_{cr}, may be calculated using elastic theory as,

$$M_{cr} = f_{cr} \left(\frac{I_g}{y_t} \right) \qquad ......(4a)$$

where

I_g = Moment of inertia of gross section, and

y_t = Distance of extreme tension fibre from neutral axis.

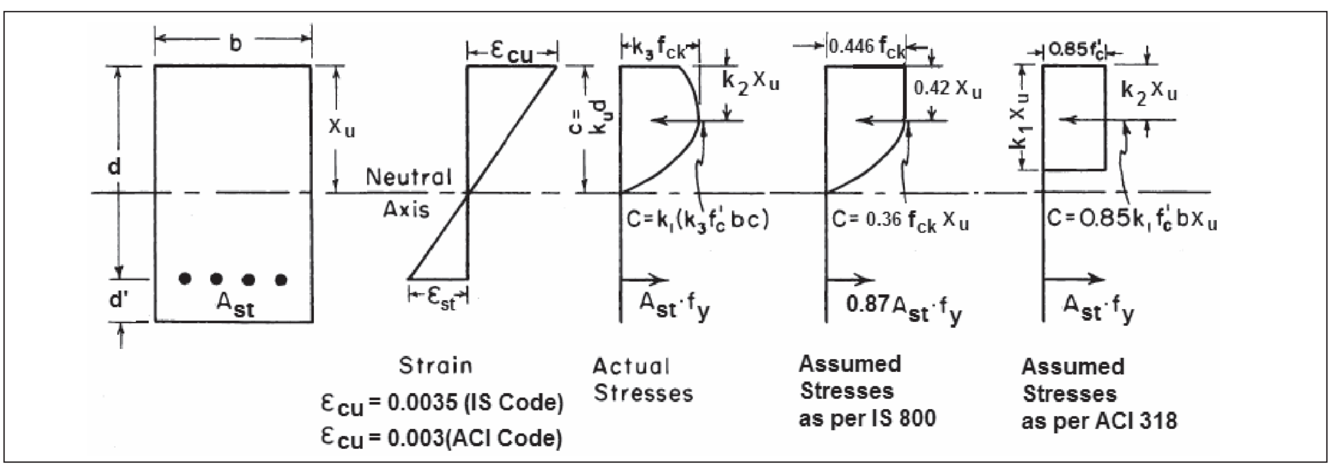

Figure 1. Strain diagram and stress blocks

Substituting the values of I_g/y_t (equal to $b_w D^2/6$, for rectangular section) and f_{cr} in Equation (4a), we get,

$$M_{cr} = 0.117 b_w D^2 \sqrt{f_{ck}} \qquad \text{......(4b)}$$

Where, D is the total depth of the beam and b_w is the width of beam for rectangular beam (For T-beams, b_w denotes the width of web).

The nominal moment of resistance as given by cracked section theory, Equation (1) without the partial safety factors, may be approximately written as

$$M_n = A_s f_y \,(d - 0.42\, X_u) \qquad \text{......(5a)}$$

The term (d - 0.42 X_u), representing the lever arm, may range from 1.00d (when steel area is zero) to 0.71d (at balanced failure). Safely assuming it to be 0.71d, we get

$$M_n = 0.71 A_s f_y\, d \qquad \text{......(5b)}$$

In rectangular beams the ratio D/d will be in the range of 0.8 to 0.95. Safely assuming it to be 1.0 in Equation (4b), and equating Equation (4b) and (5b), we get

$$0.71 A_s f_y\, d = 0.117 b_w d^2 \sqrt{f_{ck}} \qquad \text{......(6a)}$$

Rearranging the terms, we get

$$\frac{A_s}{b_w d} = \frac{0.17\sqrt{f_{ck}}}{f_y} \qquad \text{(6b)}$$

Note that the minimum steel as per the above equation is dependent on the compressive strength of concrete and hence will increase with increasing f_{ck}. But in the IS code, f_{ck} might have been assumed as 25 MPa, and the equation is given in Clause 26.5.1.1 as

$$\frac{A_s}{b_w d} = \frac{0.85}{f_y} \qquad \text{......(6c)}$$

The explanatory handbook states that this requirement will result in 0.34 percent for mild steel, thus matching the 0.3 percent minimum as required in the 1964 version of the code[2]! For cold worked deformed bars (f_y= 415 N/mm^2) it will give 0.20 percent minimum steel.

Varghese reports that in some situations, large beams designed with the minimum steel requirement of the IS code, has resulted in extensive cracking, although there are no reported failures.[3] Hence there is a need to revise the minimum tensile steel provisions of IS 456: 2000. Note that, cantilever T-beams, with their flange in tension, will require significantly higher reinforcement than specified in this clause to prevent brittle failure caused by concrete crushing; however IS 456 suggests calculating the minimum reinforcement for such T-beams, by taking b_w as the width of the web only.

It is interesting to note that the American code, till the 1995 edition, used the following equation (which is similar in format to the Indian code equation and uses a factor of safety of 2.5).[4]

$$\frac{A_s}{b_w d} = \frac{1.4}{f_y} \qquad \text{......(7a)}$$

The above equation provides a minimum tension steel of about 0.5 percent (as against the 0.3 percent minimum in the Indian code) for mild steel grade, as required by earlier editions of the ACI code. The 1995 version of the code recognized that the minimum steel as given by Equation (7a) may not be sufficient for HSC with strength greater than 35 MPa. Hence the code introduced the following equation, which has a format similar to Equation (6b).

$$\frac{A_s}{b_w d} = \frac{0.25\sqrt{f_c}}{f_y} \geq \frac{1.4}{f_y} \qquad \text{......(7b)}$$

where, f_c is the cylinder compressive strength of concrete. Equation (7b) may be rewritten in terms of cube compressive strength as below:

$$\frac{A_s}{b_w d} = \frac{0.224\sqrt{f_{ck}}}{f_y} \geq \frac{1.4}{f_y} \qquad \text{(7c)}$$

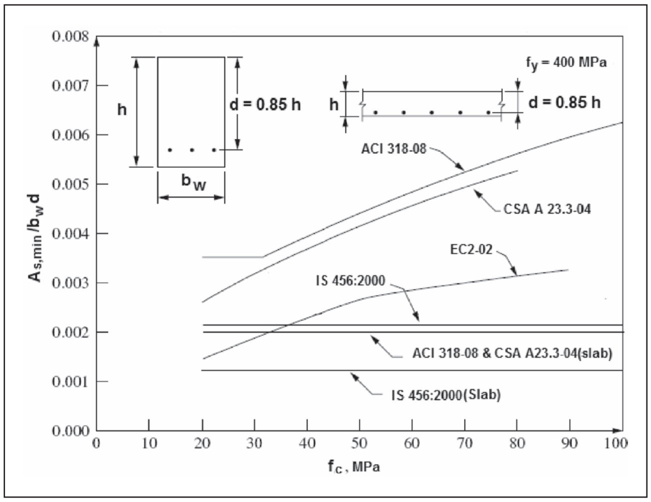

Figure 2. Comparison of minimum flexural reinforcement provisions of different codes (adapted from Ref. 16)

Note that ($0.224\sqrt{f_{ck}}$) and 1.4 are equal when f_{ck} equals 39 MPa. Hence ($1.4/f_y$) will control only when f_{ck} is less than 39 MPa. Thus for HSC, we should consider the concrete strength also, while providing minimum tensile reinforcement. It makes sense as HSC is normally brittle than NSC. In this connection, note that IS: 13920, which is used for detailing of structures subjected to seismic forces, uses the following equation which is similar to equation. (7c).[5,6]

$$\frac{A_s}{b_w d} = \frac{0.24\sqrt{f_{ck}}}{f_y} \qquad \text{......(7d)}$$

In a recent paper, Seguirant et al argued that inclusion of the ratio of yield to tensile strength of reinforcement in the equation for minimum reinforcement in flexural members will make it applicable for any grade of reinforcement, including high-strength steels.[7] (Note that high strength reinforcements with f_y = 690 MPa have recently been introduced in the market). Thus they proposed the following equation (referred as sectional provision)

$$M_n \geq \frac{1.5 f_y}{f_{su}} \frac{M_{cr}}{\phi} \qquad \text{(8a)}$$

In some cases such as T-beams with the flange in tension, the section modulus at the tension face can become quite large, resulting in substantial amount of *sectional* minimum reinforcement. Under these circumstances, the amount of minimum reinforcement can be derived directly from the applied factored load, which can be significantly smaller than the load that can theoretically cause flexural cracking. This criterion, called as *over-strength* provision, was derived by Seguirant et al as[7]

$$M_n \geq \frac{2 f_y}{f_{su}} \frac{M_u}{\phi} \qquad \text{......(8b)}$$

where M_{cr} is defined by equation 4(a), M_n is the nominal flexural resistance, as given by equation (1), M_u is the factored external moment, f_{su} is the ultimate tensile strength of reinforcement, f_y is the yield strength of reinforcement, and φ = resistant factor, and equals 0.9 in ACI code. The coefficient of 1.5 in equation (8a), normalises the ratio of yield strength to tensile strength to 1.0 for grade 415 MPa steel. The coefficient of 2.0 in equation (8b), normalises the modifier to the traditional 1.33 for grade 415 MPa steel reinforcement. Equation (8) ensures a consistent margin between the design strength and the actual strength for all grades of reinforcement. Based on Equation (8), Seguirant et al, also derived a direct, but complicated expression for the minimum reinforcement.[7]

The provisions for minimum tensile reinforcement ratio in flexural members of Indian, American, Eurocode 2, New Zealand, and Canadian codes are compared in the first row of Table 1 and Figure 2. All the codes, except the Indian code, have similar format. Hence equation (7c) or equation (8) is recommended for use in the Indian code. Note that unlike the Eurocode 2, the minimum flexure reinforcement requirements for slabs of Indian, Canadian and American Codes, are not a function of concrete strength.

It may also be interesting to note that the Bureau of Indian Standards (BIS) is proposing to revise the definition of high strength concrete in IS 456. In the Amendment 4 to be included, after discussions, in May 09, BIS has designated grades M 25 to M 60 as standard concrete (as against M25 to M55, in the current revision) and grades M 60 to M100 are designated as HSC. It may be noted that the design provisions remain unchanged (with only minor modification) from the 1978 edition of the code. These provisions were based on experiments conducted on specimens having strength up to 40 MPa. But now these provisions are extrapolated up to grade M 100, which may not be safe in certain circumstances.

An area of compression reinforcement at least equal to one-half of tension reinforcement should be provided, in order to ensure adequate ductility at potential plastic hinge zones, and to ensure that minimum of tension reinforcement is present for moment reverasal.[10,12]

Maximum flexural steel

An upper limit to the tension reinforcement ratio in flexural reinforced concrete members is also provided to avoid compression failure of concrete before the tension failure of steel, thus ensuring sufficient rotation capacity at ultimate limit state. Upper limit is also required to avoid congestion of reinforcement, which may cause insufficient compaction or poor bond between reinforcement and concrete.

For balanced section, equating tension in steel to compression in concrete at failure stage (see Figure 1), we get,

$$0.87 f_y A_{st} = 0.36 f_{ck} b x_u \quad \text{......(9a)}$$

This can be rewritten as,

$$\frac{A_{st}}{bd} = \frac{0.36 f_{ck}}{0.87 f_y}\left(\frac{x_u}{d}\right) \quad \text{......(9b)}$$

The above equation is rewritten, in terms of percentage of steel $p_t = \left(\frac{A_{st}}{bd}\right)100$, as

$$p_t = 41.38 \frac{f_{ck}}{f_y}\left(\frac{x_u}{d}\right) \quad \text{......(10)}$$

IS 456 limits the values of (x_u/d) in order to avoid brittle failure, by stating that the steel strain ε_{cu} at failure should not be less than the following:

$$\varepsilon_{su} = \frac{f_y}{1.15 E_s} + 0.002 = \frac{0.87 f_y}{E_s} + 0.002 \quad \text{......(11)}$$

Table 1. Comparison of provisions of different Codes[1,4,8-10]

Requirement	Code provision as per				
	IS 456	ACI 318**	CSA A23.3**	Eurocode2*	NZS 3101**
Minimum tensile steel for flexure+, $\frac{A_s}{b_w d} \geq$	$\frac{0.85}{f_y}$ For T-sections use b_w only.	$\frac{0.224\sqrt{f_{ck}}}{f_y} \geq \frac{1.4}{f_y}$ For T-sections, use $2b_w$ or b_f whichever is smaller	$\frac{0.18\sqrt{f_{ck}}}{f_y}$ For T- beams b_w is taken in the range $1.5 b_w$ to $2.5 b_w$	$\frac{0.26 f_{ctm}}{f_y} \geq 0.0013$ For T-beams b_w is taken as mean breadth.	$\frac{0.224\sqrt{f_{ck}}}{f_y}$ For T-beams b_w is taken smaller of 2 b_w or width of flange.
Maximum tensile steel for flexure, ≤	0.04bD	Net tensile strain in extreme tensile steel ≥ 0.005	Tension reinforcement limited to satisfy $\frac{c}{d} \leq \frac{700}{700 + f_y}$	0.04bD	$\frac{0.9 f_{ck} + 10}{6 f_y} \leq 0.025$
Minimum shear reinforcement, $\frac{A_s}{b_w s_v} \geq$	$\frac{0.4}{0.87 f_y}$ When $\tau_v > 0.5\tau_c$	$\frac{0.9\sqrt{f_{ck}}}{16 f_y} \geq \frac{0.33}{f_y}$ When applied shear is greater than 0.5 X concrete strength	$\frac{0.054\sqrt{f_{ck}}}{f_y}$ When applied shear is greater than concrete strength	$\frac{0.08\sqrt{f_{ck}}}{f_y}$ When applied shear is less than shear strength of concrete	$\frac{0.9\sqrt{f_{ck}}}{16 f_y}$ When applied shear is greater than 0.5 X concrete strength
Spacing of Minimum Stirrups ≤	0.75 d ≤ 300mm	0.5 d ≤ 600 mm & 0.25 d ≤ 300 mm, when $V_s > \sqrt{f_c} b_w d/3$	0.63 d ≤ 600 mm 0.32 d ≤ 300 mm When $V_u > \phi_c f_c b_w d/8$	0.75 d ≤ 600 mm	0.5 d ≤ 600 mm 0.25 d ≤ 300 mm, when $V_s > \sqrt{f_c} b_w d/3$

** The cylinder strength is assumed as equal to 0.8 times the cube strength.
+ Alternatively the ultimate flexural strength should be at least one third greater than the factored moment
f_{ctm}= Mean axial tensile strength = $0.30 (f_{ck})^{0.666}$
b_f = breadth of flange; b_w = breadth of web

From the similar triangles of the strain diagram of Figure 1, we get

$$\frac{x_u}{d} = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{su}} = \frac{0.0035}{0.0035 + \varepsilon_{su}} \qquad ......(12)$$

Substituting the various values of ε_{su} for different values of steel, and using E_s = 200 x 10^3 N/mm^2, we get the maximum limiting values of (x_u/d), as shown in Table 2.

Table 2. Limiting values of x_u/d

Steel grade, f_y (MPa)	Yield strain, ε_{su}	$(x_u/d)_{limit}$
250	0.0031	0.530
415	0.0038	0.479
500	0.0042	0.455

Substituting the above values of (x_u/d) in Equation (10) we may get the limiting percentage of steel, for various steel grades as per Table 3.

Table 3 Limiting steel percentage for limiting values of x_u/d

Steel grade, f_y (MPa)	$(x_u/d)_{limit}$	$p_t (f_y/f_{ck})$
250	0.530	21.93
415	0.479	19.82
500	0.455	18.82

Until 2002, the ACI code permitted p_t values up to 75 percent of the steel required for balanced sections, as the maximum flexural reinforcement. Using this rule and selecting M25 and grade 415 steel, we get maximum percentage of steel =0.75 x 19.82 x 25/415 = 0.89. But IS 456 stipulates that the maximum percentage of tension reinforcement in flexural members as 4 percent, which is very high.[1] Note that IS 13920 suggests a percentage of steel of 2.5 percent, which is also high.[5]

Although the American code specified the maximum percentage of steel as 75 percent of balanced reinforcement ratio in the earlier versions, in the 2002 version of the code, the provision was changed, as it may become complicated for flanged sections, and sections that use compression reinforcement. In the present edition of the code the ductility of the section is controlled by controlling the tensile strain, ε_t, in the extreme layer of tensile steel (see Figure 3).[4,11] Thus, when the net tensile strain in the extreme tension steel, ε_t, is equal

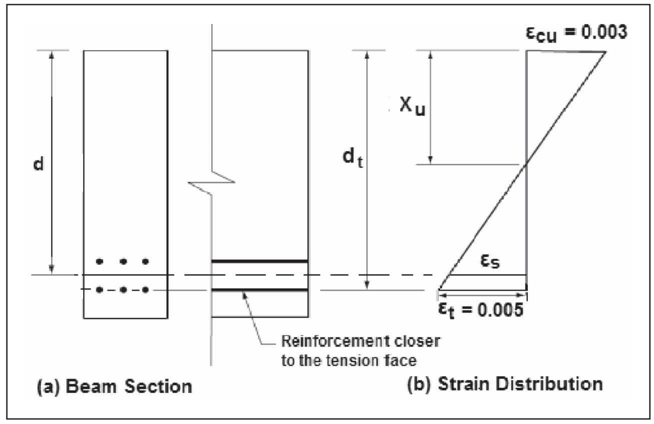

Figure 3. Definition of Tension-controlled sections in ACI 318 code

to or greater than 0.005, and the concrete compressive strain reaches 0.003, the section is defined as tension-controlled (Sections with ε_t less than 0.003 are considered compression controlled and not used in singly reinforced sections; Sections with ε_t in the range of 0.003 to 0.005 are considered as transition between tension and compression controlled).[4,11] Such a tension-controlled section will give ample warning of failure with excessive deflection and cracking. For Grade 415 steel, the tensile yield strain is ε_y = 415/ (200 x 10^3) =0.00208. Thus the tension-controlled limit strain of 0.005 was chosen to be 2.5 times the yield strain. Such tension-controlled sections will result in a moment-curvature diagram similar to that shown in Figure 4 (the one with area of reinforcement equal to 2900 mm^2).

Note that in the ACI code different strength reduction factors (called φ factors) are used- ranging from 0.9 (tension controlled) to 0.65 (compression controlled) - to calculate the design strength of members from the calculated nominal strength. Also flexural members are usually chosen as tension-controlled, whereas compression members are usually chosen as compression-controlled. The net tensile strain limit of 0.005 for tension-controlled sections was chosen to be a single value that applies to all types of steel (prestressed and non-prestressed).[11]

From similar triangles of Figure 3, we may deduct that for tension controlled flexural members, x_u/d = 3/8. Substituting this value in Equation (10), we get

$$p_t = 15.5 \frac{f_{ck}}{f_y} \leq 2.5 \qquad ......(13)$$

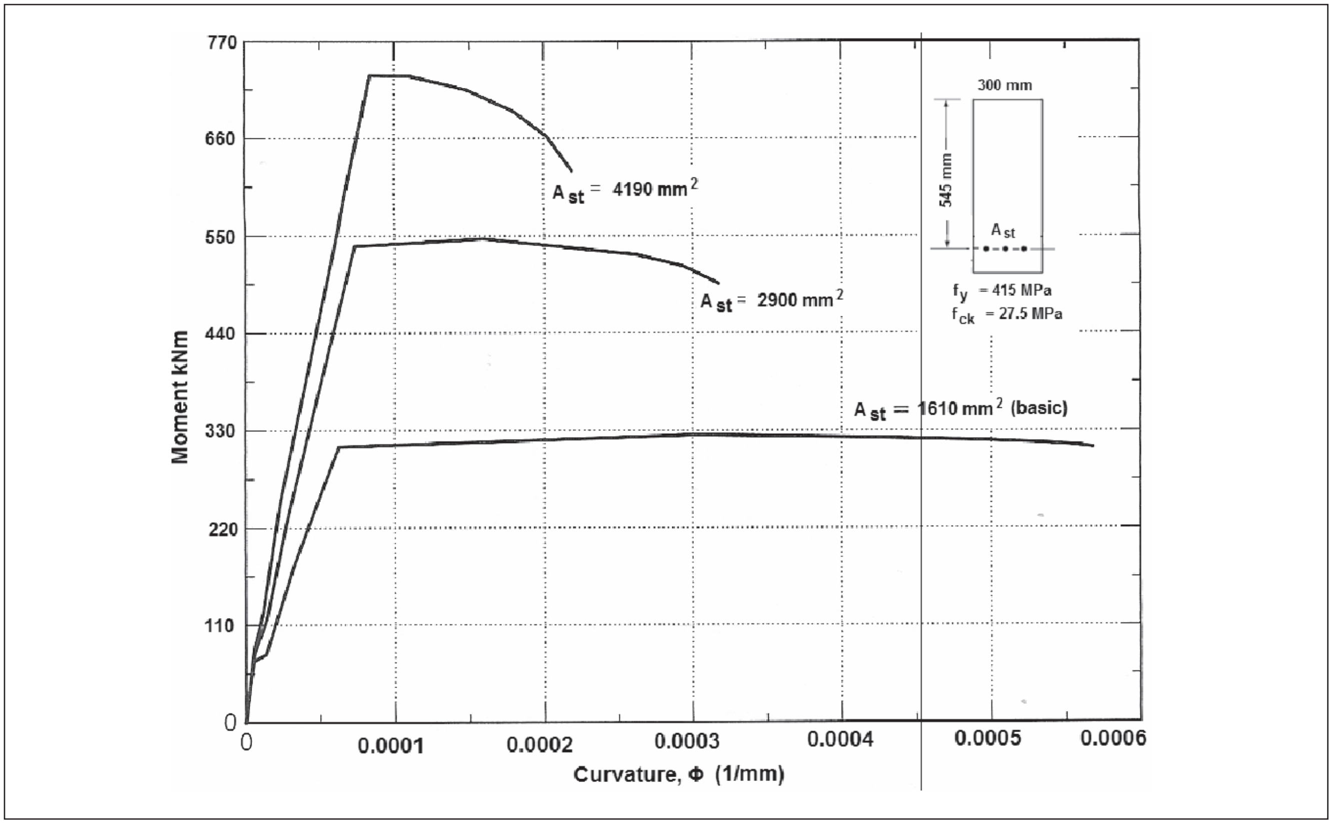

Figure 4. Moment-curvature diagram of beam with varying steel area (Adapted from Ref.12)

For M25 concrete and grade 425 steel, we get p_t =0.93 percent, which is comparable to 0.89 percent obtained earlier using the rule specified in the older version of ACI (i.e. 75 percent of steel required for balanced section).

The provisions for maximum tensile reinforcement in flexural members of Indian, Eurocode 2, American, New Zealand, and Canadian codes are compared in the second row of Table 1. Except the Indian code and Eurocode2, all the other codes have similar format and involve both f_{ck} and f_y. Hence Equation (10) is suggested for use for specifying maximum tension steel in IS 456.

Minimum shear reinforcement

When the principal tensile stress within the shear span exceeds the tensile strength of concrete, diagonal tension cracks are initiated in the web of concrete beams. These cracks later propagate through the beam web, resulting in brittle and sudden collapse, when web reinforcement is not provided (The diagonal cracking strength of reinforced concrete beams depends on the tensile strength of concrete, which in turn is related to its compressive strength). Hence minimum shear reinforcements are often stipulated in different codes.

When shear reinforcement are provided, they restrain the growth of inclined cracking. Ductility is also increased and a warning of failure is provided. Such reinforcement is of great value if a member is subjected to an unexpected tensile force due to creep, shrinkage, temperature, differential settlement, or an overload.

It is interesting to note that the shear provisions of the ACI code were revised after the partial collapse of Wilkins Air Force Depot in Shelby, Ohio, in 1955.[13] At the time of collapse, there were no loads other than the self-weight of the roof. The 914 mm deep beams of this warehouse- the concrete alone (with no stirrups) was expected to carry the shear forces- and had no shear capacity once cracked. The beams had 0.45 percent of longitudinal reinforcement.[13] The beams failed at a shear stress of only about 0.5 MPa, whereas the ACI Code (1951 version) at the time permitted an allowable working stress of 0.62 MPa for the M20 concrete used in the structure. Experiments conducted at the Portland Cement Association (PCA) on 305 mm deep model beams indicated that the beams could resist a shear stress of about 1.0 MPa prior to failure.[13] However, application of an axial tensile stress of about 1.4 MPa reduced the shear capacity of the beam by 50 percent. Thus, it was concluded that tensile stresses caused by the restraint of shrinkage and thermal movements caused the beams of Wilkins Air Force Depot to fail at such low thermal shear stresses.[13] This failure outlines the importance of providing minimum shear reinforcement in beams. It has to be noted that repeated loading will result in failure loads which may be 50 to 70 percent of static failure loads.[14]

The shear behaviour of beams with stirrups is normally evaluated by the truss theory developed by Moersch in 1912.[3] Thus the reinforced concrete beam is considered as a truss with the following components: compression concrete constituting the top chord, the tensile reinforcement forming the bottom chord, stirrups acting as vertical web tension members, and the pieces of concrete between the approximately 45^o tension diagonal cracks, acting as diagonal compression members of the web. The design of stirrups is usually based on the vertical component of diagonal tension, while the horizontal component is resisted by the longitudinal tensile steel of the beam. If we consider a 2-legged stirrup with a total area of legs as A_{sv}, spaced at s_v, crossing a crack line at 45^o,

The number of stirrups crossed by the crack = d/s_v ......(14a)

Shear resistance of the vertical stirrups,
$V_s = 0.87 f_y A_{sv}(d/s_v)$ ......(14b)

The above equation may also be written as

$$\frac{A_{sv}}{s_v} = \frac{(\tau_v - \tau_c)b_w}{0.87 f_y} \qquad \text{......(14c)}$$

where

A_{sv}= Total cross sectional area of stirrup legs effective in shear,

s_v = Stirrup spacing along the length of the member

τ_v= calculated nominal shear stress ($V_u / b_w d$), MPa

τ_c = design shear strength of concrete, MPa, and

$V_s = V_u - V_c = (\tau_v-\tau_c)b_w d$

V_u = Applied shear force due to external loads

V_c = Shear strength provided by concrete

The other terms are defined already.

As per clause 26.5.1.6 of the IS 456:2000, minimum shear reinforcement should be provided in all the beams when the calculated nominal shear stress τ_v is less than half of design shear strength of concrete, τ_c, as given in Table 19 of the code. The minimum stirrup to be provided is given by the following equation.

$$\frac{A_{sv}}{b_w s_v} \geq \frac{0.4}{0.87 f_y} = \frac{0.46}{f_y} \qquad \text{......(15)}$$

Note that the code restricts the characteristic yield strength of stirrup reinforcement to 415 N/mm^2. Comparing Eqns (14c) and (15), we get, $(\tau_v-\tau_c)$ = 0.40 MPa. This shows that the amount of required minimum stirrups corresponds to a nominal shear stress resisted by stirrups of 0.40 MPa (The Joint ASCE-ACI committee on shear recommended 0.34 MPa).[14]

As per IS 456, for vertical stirrups, the maximum spacing of shear reinforcement shall not exceed 0.75 d or 300 mm, which ever is less. Note that the IS code limits the maximum yield strength of web reinforcement to 415 N/mm^2, to avoid the difficulties encountered in bending high strength stirrups(they may be brittle near sharp bends) and also to prevent excessively wide inclined cracks.

Till the 2002 version, The ACI code used a formula similar to that given in the Indian code, with a coefficient equal to (1/3) instead of 0.46; thus the requirement for minimum area of transverse reinforcement was independent of the concrete strength. Tests conducted by Roller and Russell on HSC beams indicated that the minimum area of shear reinforcement is also a function of concrete strength.[15] Hence the current version of ACI code provides the following equation for minimum shear reinforcement.

$$\frac{A_{sv}}{b_w s_v} = \frac{0.9\sqrt{f_{ck}}}{16 f_y} \geq \frac{1}{3 f_y} \qquad \text{......(16)}$$

Note that the above equation provides for a gradual increase in the minimum area of transverse reinforcement, while maintaining the previous minimum value. In seismic regions, web reinforcement is required in most beams, because the shear strength of concrete is taken equal to zero, if earthquake induced shear exceeds half the total shear.[12]

Stirrups will not be able to resist shear unless an inclined crack crosses them. For this reason ACI code section

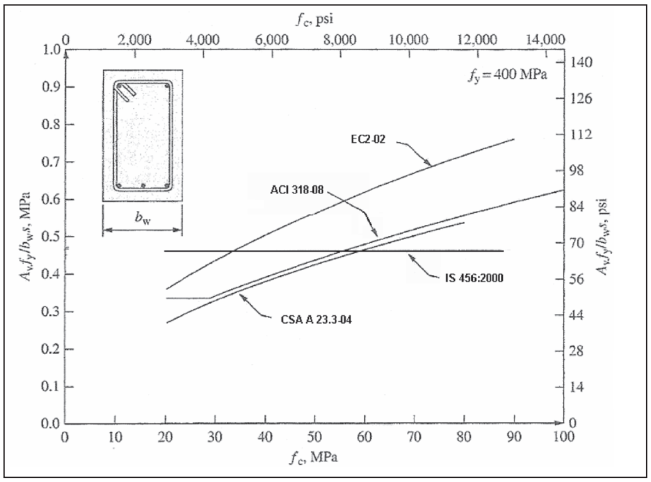

Figure 5. Minimum shear reinforcement as a function of f_c as per different codes (adapted from Ref. 16)

11.4.5.1 sets the maximum spacing of vertical stirrups as the smaller of d/2 or 600 mm, so that each 45^o crack will be intercepted by at least one stirrup. If V_u/ϕ-V_c exceeds $\sqrt{f_c} b_w d/3$, the maximum allowable stirrup spacing is reduced to half of the above mentioned spacing. Thus for vertical stirrups, the maximum spacing is the smaller of d/4 or 300 mm. The above stipulation is due to the following: closer stirrup spacing leads to narrower inclined cracks and also will provide better anchorage for the lower ends of the compression diagonals.[12] IS 13920 also adopts a spacing of stirrups as d/4 or 8 times the diameter of the smallest longitudinal bar, but not less than 100 mm at the ends of beam over a length of 2d (in plastic hinge regions) and a spacing of d/2 elsewhere.[5] The ACI code also restricts maximum yield strength of web reinforcement to 415 N/mm^2, although the New Zealand code allows design yield strength up to 500 MPa. Based on the above discussions, equation (16) is proposed for the Indian code with the spacing as stipulated in IS 13920.

The provisions for minimum shear reinforcement in flexural members of Indian, Eurocode 2, American, New Zealand, Canadian codes and compared in the third and fourth rows of Table 1 and Figure 5. Except the Indian code, all the other codes have similar format and consider both f_{ck} and f_y.

Upper limit on area of shear reinforcement

If the area of shear reinforcement is large, failure may occur due to the shear compression failure of concrete struts of the truss prior to the yielding of steel shear reinforcement. Hence, an upper limit to the area of shear reinforcement corresponds to the yielding of shear reinforcement and shear compression failure of concrete simultaneously, is necessary. Based on this, the maximum shear force carried by the beam is limited. IS 456 recommends that this value should not exceed $\tau_{uc,max}$ given by (See Table 20 of IS 456)[2]

$$\tau_{uc,max} = 0.85 \times 0.83 \sqrt{f_c} = 0.631\sqrt{f_{ck}} \quad ......(17)$$

Recently Lee and Hwang compared the test results of 178 RC beams reported in the literature and the 18 beams tested by them and found that the shear failure mode changes from under-reinforced to over-reinforced shear failure when $p_s f_y / f_c$ is approximately equal to 0.2. Hence they suggested the maximum amount of shear reinforcement for ductile failure as given below[17]

$$p_{smax} = 0.2\ (f_c / f_y) \quad ......(18a)$$

In terms of f_{ck} , the above equation may be written as

$$p_{smax} = 0.16\ (f_{ck} / f_y) \quad ......(18b)$$

where $p_{smax} = A_v/(s_v b_w)$

Lee and Hwang also found that the amount of maximum shear reinforcement, as suggested by ACI 318-08, and given in Equation (19) need to be increased for high strength concrete beams, as test beams with greater than 2.5 times the p_{smax} given by Equation (19), failed in shear after yielding of the stirrups.[17]

$$p_{smax} = 2\sqrt{f_c}/(3 f_y) \quad ......(19)$$

The expressions suggested by Canadian and Euro code are more complicated but found to agree with the test results reasonably.[17] But these equations for maximum shear reinforcement are proportional to concrete compressive strength, whereas the Indian and American code equations are proportional to the square root of concrete compressive strength. It is also interesting to note that the Canadian and Eurocode equations are based on analytical methods such as the variable angle truss method, where as the Indian and American code equations are based on experimental results. Based on the above, the expression presented in equation (18b) is suggested for the Indian code.

Maximum diameter of longitudinal beam bars

The New Zealand code also restricts the maximum diameter of longitudinal bars passing through beam-column joints in a ductile structure, in order to prevent premature slipping of the bar. When the critical load combination for flexure in a beam, at the face of an internal column, includes earthquake actions, the bar diameter is controlled by the equation (assuming that the average bond stress is limited to a maximum value of 1.5 $\alpha_f \sqrt{f_c}$):

$$\frac{d_b}{h_c} = 4\alpha_f \frac{\sqrt{f_c}}{f_y} \qquad ......(20)$$

Where d_b is the diameter of bar, h_c is the column depth and α_f is taken as 0.85 where the beam passes through a joint in a two-way frame and as 1.0 for a joint in a one-way frame.

Summary and conclusions

The minimum and maximum limits on longitudinal and transverse reinforcement ratios provided in the Indian code are found to depend only on the yield strength of reinforcement and independent on the concrete strength. Moreover the extrapolation of these provisions, which were derived for ordinary concrete grades up to M40, to high strength concrete flexural members, may result in compression failure of concrete rather than the desired ductile failure of steel reinforcement. They also may not protect the high strength flexural members from over loads or from actions due to differential settlement, creep, shrinkage or thermal movements which may create additional tensile forces. Hence the Indian code provisions are compared with the ACI code provisions and also with the provisions of Eurocode 2, Canadian and New Zealand codes, and based on these, suitable modifications to the expressions are suggested for future editions of the Indian code. As more than 65 percent of the area of our country falls under Zone III or above as per the recent revision of IS 1893, these modifications assumes greater importance as they are intended to induce ductile behaviour.

References

1. ______*Indian Standard code of practice for plain and reinforced concrete*, IS 456: 2000, Bureau of Indian Standards, New Delhi, July 2000, p. 100.
2. ______*Explanatory handbook on Indian Standard code of practice for plain and reinforced concrete*, SP :24 (S&T), Bureau of Indian Standards, 1984, New Delhi, pp. 164.
3. Varghese, P.C. *Limit States Design of Reinforced Concrete*, 2006, Second Edition, Prentice –Hall of India Ltd., New Delhi, 545pp.
4. ______ *Building Code Requirements for Structural Concrete and Commentary*, ACI 318M-2008, American Concrete Institute, Farmington Hills, Michigan, 2008, p. 473.
5. _____*Indian Standard code of practice for ductile detailing of reinforced concrete structures subjected to seismic forces*, IS 13920:1993, Bureau of Indian Standards, New Delhi, p. 14 (also see http://www.iitk.ac.in/nicee/IITK-GSDMA/EQ11.pdf for the draft revised edition of code).
6. Medhekar, M.S., and Jain, S.K., Proposed minimum reinforcement requirements for flexural members, *The Bridge and Structural Engineer, ING-IABSE*, June 1993, Vol. 23, No. 2, , pp. 77-88.
7. Seguirant, S.J., Brice, R. and Khaleghi, B., A comparison of existing and proposed minimum flexural reinforcement requirements for reinforced concrete members, *Structural Engineering and Design,* July 2010, Vol. 11, No. 6, pp. 26-30.
8. _____ *Eurocode 2: Design of concrete structures Part 1-1: General rules and rules for buildings*, EN 1992-1-1:1992, European Committee for Standardization (CEN), Brussels, Dec. 1991.
9. _____ *Design of concrete structures,* CSA A23.3-2004, Canadian Standards Association, Ontario,2004, 232, pp. (www.csa.ca)
10. _____NZS 3101:2006, Part 1: The design of Concrete structures, and Part 2: Commentary, Standards New Zealand, Wellington, 2006 (www.standards.co.nz).
11. Mast, R.F., Unified design provisions for reinforced and prestressed concrete Flexural and compression members, *ACI Structural Journal*, March-April 1992, Vol. 89, No. 2, pp. 185-199.
12. Wight, J.K., and MacGregor, J.G., *Reinforced Concrete: Mechanics and Design*, 2009, Fifth Edition, Pearson Prentice Hall, New Jersey, p. 1112.
13. Feld, J. and Carper, K., *Construction Failure*s, 1997, Second Edition, Wiley-Interscience, New York, pp. 528.
14. ACI-ASCE Committee 426, Shear Strength of Reinforced Concrete Members, *Proceedings*, *ASCE, Journal of the Structural Div*., June 1973, Vol. 99, No. ST6, pp. 1091-1187.
15. Roller, J.J., and Russell, H.G., Shear strength of high-strength concrete beams with web reinforcement, *ACI Structural Journal*, March-April 1990, Vol. 87, No. 2, pp. 191-198.
16. Li, Z., and Zhang, Y., High-performance Concrete, *Handbook of Structural Engineering*, 2005, Second Edition, W.F. Chen and E.M. Lui (eds.), CRC Press, Boca Raton, FL., p. 1768.
17. Lee, J.Y., and Hwang, H.B., Maximum shear reinforcement of reinforced concrete Beams, *ACI Structural Journal*, September-October 2010, Vol. 107, No. 5, pp. 580-588.

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